الأحد ٢٠١٥/٧/٢٦	٣٤٢ – نظرية أعداد (الامتحان الثاني)	الجامعة الأردنية
الرقم الجامعي:	Key	اسم الطالب:

[1] Show that any integer of the form $3n+2$ has a prime factor of the same form.

Let $m \in \mathbb{N}$, $m = 3n+2$

If all prime factors of m are of the form $3k$

Then $m = \prod p_i = \prod 3k_i \neq 3n+2$, a contradiction

If all prime factors of m are of the form $3k+1$

Then $m = \prod p_i = \prod (3k_i+1) \neq 3n+2$, a contradiction

If m has no prime factors of the form $3k+2$, but of the forms $3k$ and $3k+1$

Then $m = \prod p_i = \prod (3k_j+1) \prod 3n_\ell \neq 3n+2$, a contradiction

Thus m must have a prime factor of the form $3k+2$.

[2] Solve the Diophantine equation $24x + 138y = 18$

$138 = 24(5) + 18$

$24 = 18(1) + 6$

$18 = 6(3) + 0$

$\gcd(138, 24) = 6 \mid 18$

$6 = 24 - 18$

$= 24 - (138 - 24(5))$

$6 = 24(6) + 138(-1)$

$18 = 24(18) + 138(-3)$

$x_0 = 18 \wedge y_0 = -3$

$x = 18 + \frac{138}{6} t = 18 + 23t$

$y = -3 - \frac{24}{6} t = -3 - 4t$

13	15	132965	1
14	14	130799	2
7	4	137862	3
15	15	132267	4
17	13	145573	5
16	10	138378	6
17	20	133006	7
5	1	2120077	8
12	16	121232	9
10	4	2120572	10
6	14	133396	11
12	10	131189	12
10	4	132774	13
13	16	131672	14
14	20	111156	15
16	16	107866	16
7	1	2110238	17
15	15	121244	18
14	11	139079	19
8	3	119935	20
17	10	127711	21
15	18	121247	22
7	5	131569	23
4		2121034	24
19	16	133510	25
	2	131396	26
17	12	132925	27
15	17	121255	28
9	3	130511	29
15	15	134151	30
16	16	128068	31
11	11	124256	32
15	12	133463	33
16	14	133146	34
4	8	121274	35
13	12	133001	36
15	15	138372	37
6	2	125408	38
	0	132776	39
17	20	133107	40
	15	121286	41
17	13	131826	42
12	18	133654	43
15	12	2110208	44
15	17	133554	45
9	15	133575	46
15	13	133021	47
16	16	129228	48

7	9	121663	49
11	3	132505	50
14	9	132104	51
16	16	132719	52
15	5	121300	53
13	10	132786	54
16	19	127593	55
11	12	129352	56
7	4	125413	57

[3] Show that if $a, b \in \mathbb{Z} - \{0\}, k > 0$, then $\gcd(ka, kb) = kgcd(a, b)$.

[4] Show that there are infinite primes ending with the block 321.

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[1] Let $a \in \mathbb{N} - \{3\}$. Show that $a, a+2, a+4$ cannot be all prime.

$a = 3k + r \qquad 0 \le r < 3$

if $r = 0$, $\quad a = 3k$, since $a \neq 3$, $3 \mid a$, a is composite.

if $r = 1$ $\quad a+2 = 3k+3 = 3(k+1)$, $3 \mid a+2$, $a+2$ is composite

if $r = 2$ $\quad a+4 = 3k+6 = 3(k+2)$, $3 \mid a+4$, $a+4$ is composite

[2] Solve the Diophantine equation $56x + 72y = 40$

$72 = 56(1) + 16$

$56 = 16(3) + 8$

$16 = 8(2) + 0$

$\gcd(72, 56) = 8 \mid 40$

$8 = 56 - 16(3)$

$\quad = 56 - 3(72 - 56)$

$8 = 56(4) + 72(-3)$

$40 = 56(20) + 72(-15)$

$x_0 = 20 \quad \wedge \quad y_0 = -15$

$x = 20 + \frac{72}{8} t = 20 + 9t$

$y = -15 - \frac{56}{8} t = -15 - 7t$

[3] Show that if $a, b \in \mathbb{Z} - \{0\}$, then $lcm(a,b) = \frac{ab}{\gcd(a,b)}$

[4] Show that there are infinite primes ending with the block 123.

Let $a = 123$ $b = 1000$

$\gcd(a,b) = 1$

By Dirichelt Thm $123 + 1000,\ 123 + 2(1000),\ 123 + 3(1000), \ldots$

$= 1123,\ 2123,\ 3123, \ldots$

Contains infinitely many primes each ending with the block 123.